Game Theory

9 – Games of Incomplete information:

dynamic games with no updating of players' beliefs

Review of lecture eight

- Definitions of static games of incomplete information
- Definition of player’s types
- Bayesian games and how to find a Bayesian Nash Equilibrium

Incomplete information and extensive form

- Even in some dynamic setting, we can have different **types** but NO possibility to **update** beliefs by players according to actions undertaken during the game **whenever** the "type players" do not move FIRST!

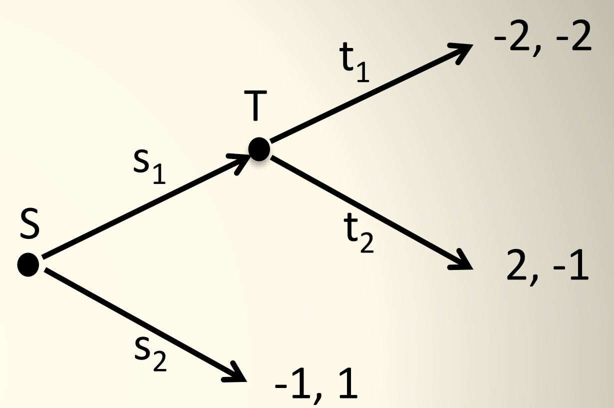
Incomplete information and extensive form

- Let us consider an electoral college where plurality rule is in effect
- In a multi-partisan system, party T is the usual winner presenting a popular local candidate
- Considering the opportunity to interrupt this tradition, party S may choose to present one of its national leader (s_1) or to give up (s_2)
- In case (s_1) is played, T may decide to answer presenting itself a national leader (t_1) or to continue with the local candidate (t_2)

"Choosing the candidate" game (1)

For **T** the best outcome is the **status quo** (it wins with no costs)
Behaving **as usual** against the challenge is the second (the contest is uncertain but a national leader is saved for other situations)
The last result is **accepting the challenge** (the result is uncertain but a national leader is lost for other situations)

For **S** the best result is **challenging** with no reaction by T (high probability of winning)
The second best is **not to challenge** at all (it loses but the national leader is saved
The last result is when its **challenge is accepted** (no better chance to win and one national leader lost)



The solution it immediate by backward induction:

- SPNE (s_1 ; t_2)
- The game develops with a challenge ignored
- The outcome is (2,-1)

“Choosing the candidate” game (2)

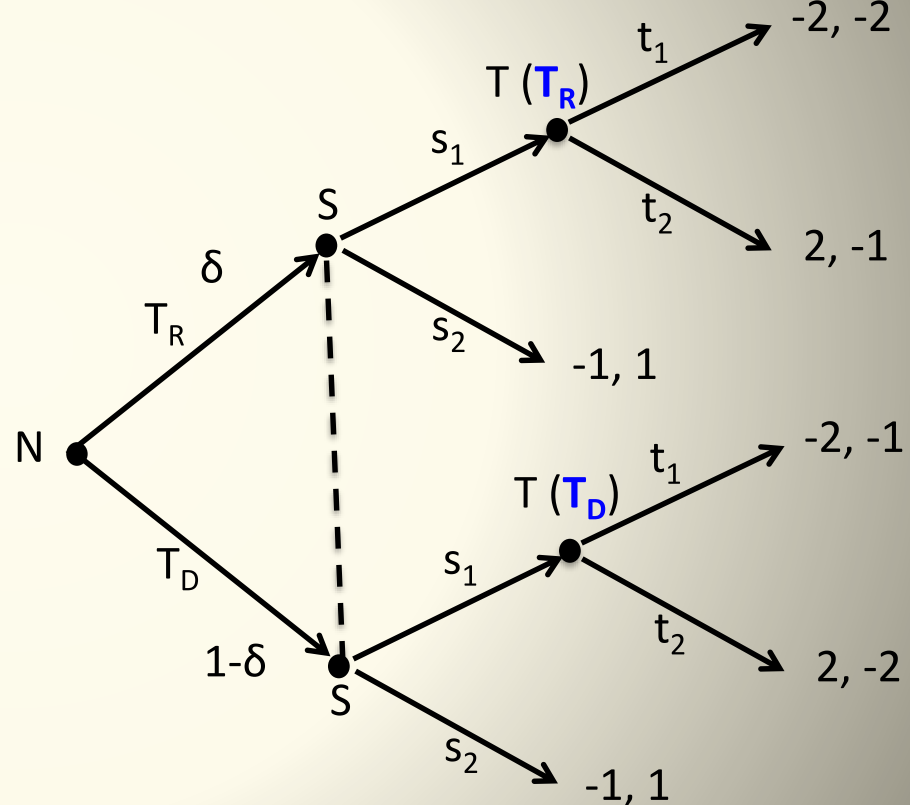
Let us **now** suppose that the challenging party S is uncertain about party T preferences

1. T is considered **preferring to lose** the college than to spend a national leader
2. Or T is considered **preferring to win** the college than to go without a leader in other competitions

In other words T may be represented by **the type T_R** (ready to loose the college) or by **the type T_D** (determined to get the seat)

Uncertainty can be represented by an **initial move of the Nature** choosing the type T_R (with probability delta δ) or T_D (with probability 1- δ) of party T

N's move is private information of player T

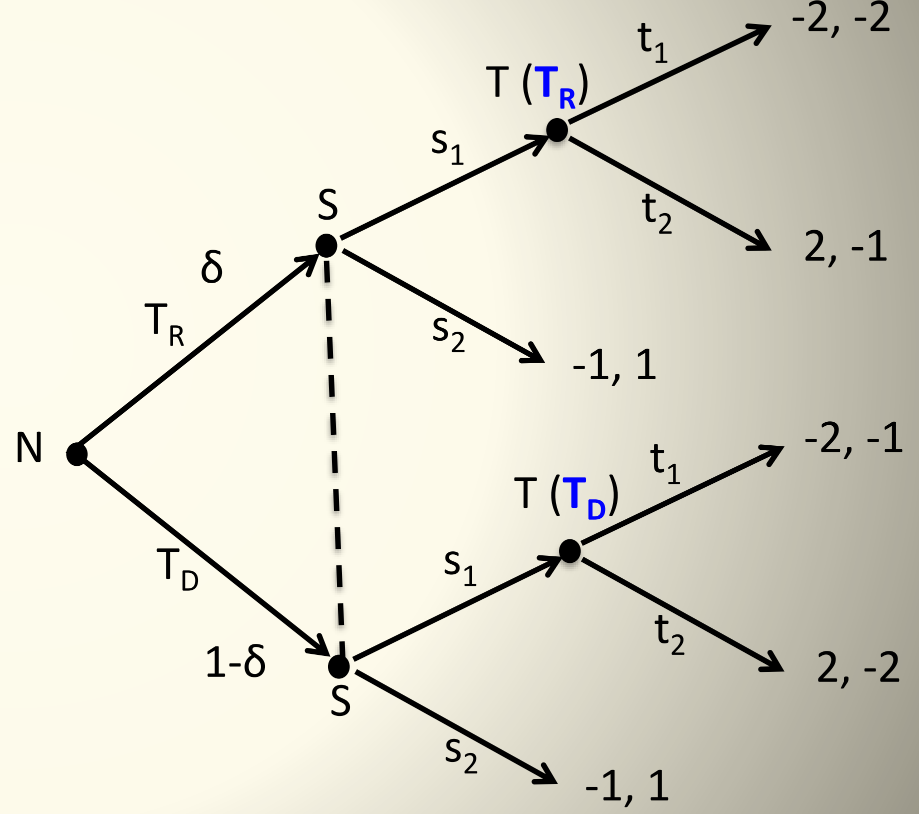


"Choosing the candidate" game (2)

The game is among two players (S and T), BUT three types of players: S, T_R and T_D

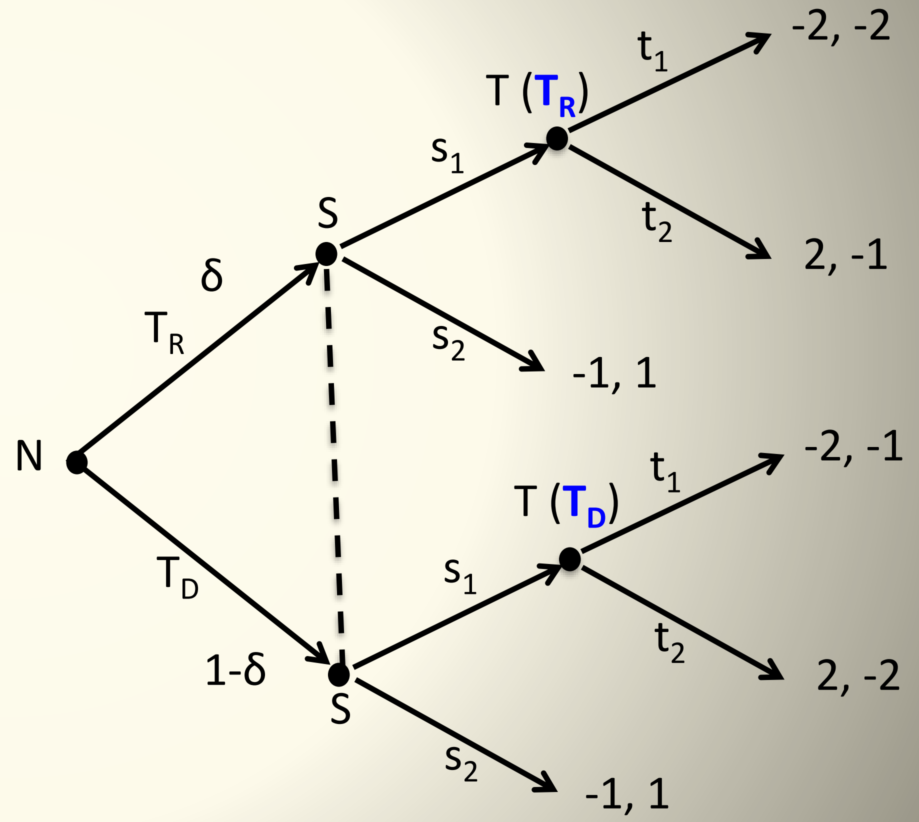
How many subgames do you have? How many strategies are available for each type of player?

Each of the two t type is involved in one proper subgames, therefore we can apply **backward induction**!



"Choosing the candidate" game (2)

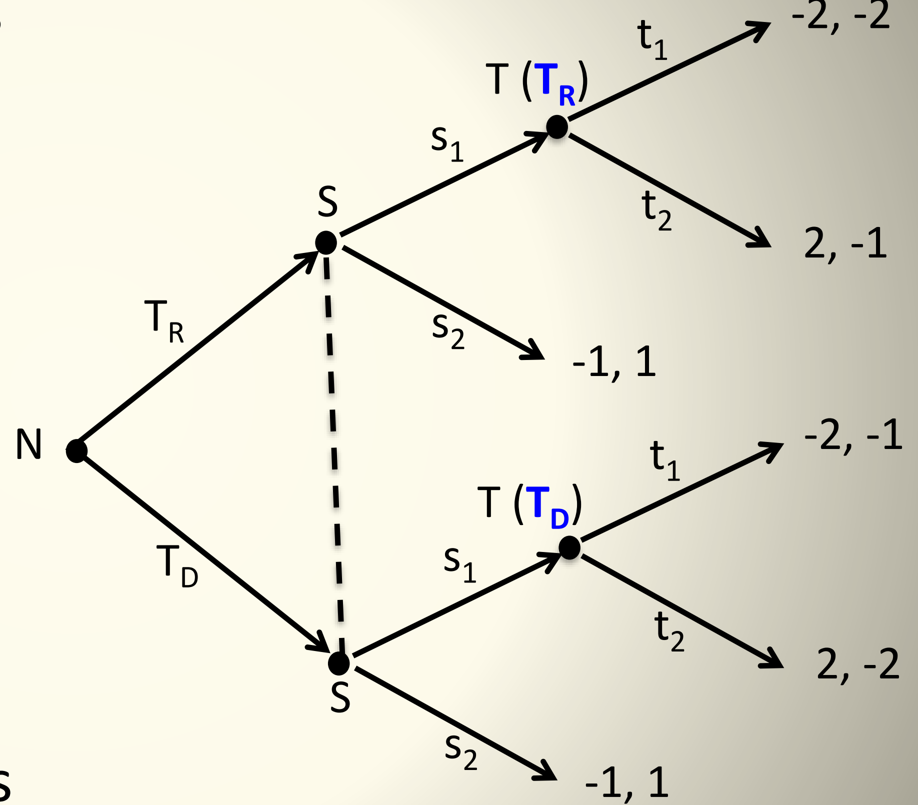
Let's solve the game by applying first the already discussed conditions ***[2]*** and ***[3]*** in the case of a static Bayes game



"Choosing the candidate" game (3)

There are eight strategy profiles that are:
(s_1,t_1,t_1); (s_1,t_1,t_2); (s_1,t_2,t_1); (s_1,t_2,t_2); (s_2,t_1,t_1); (s_2,t_1,t_2); (s_2,t_2,t_1); (s_2,t_2,t_2)

However, playing t_1 for T_R or t_2 for T_D wouldn't be consistent with backward induction (conditions ***[2]*** and ***[3]!!!***)
Therefore…only the two profiles **(s_1,t_2,t_1)** and **(s_2,t_2,t_1)** survive



“choosing the candidate” game (4)

Let’s go back to player S and let’s apply condition ***[1]***. In this case the probabilities δ that S assigns to T_R and $1-\delta$ that S assigns to T_D are unknown

We need to see if values exist of δ such that one or the other survived profiles **(s_1,t_2,t_1)** and **(s_2,t_2,t_1)** are not dominated for S

The expected utilities of party S for the two profiles are

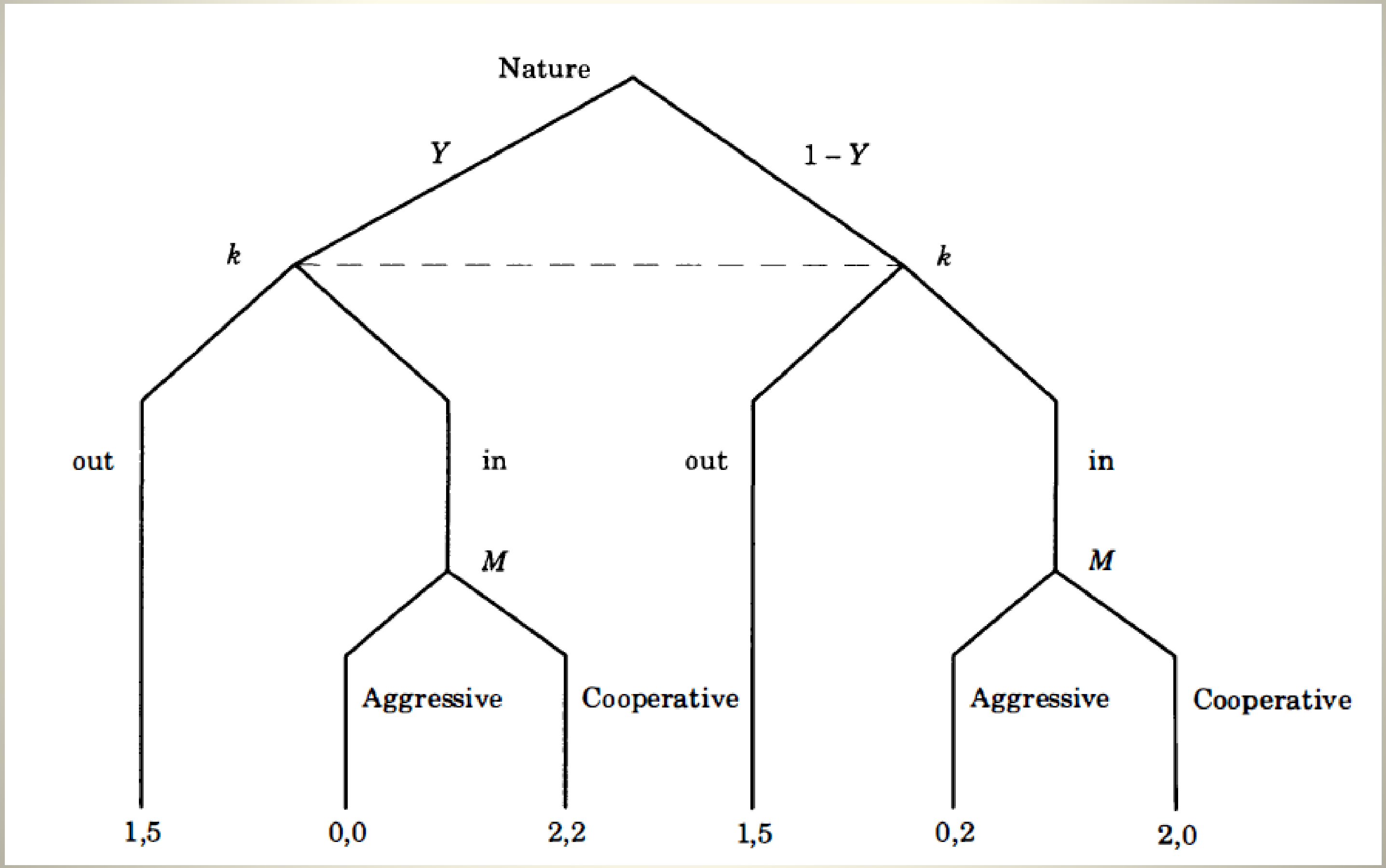
$$Eu_s(s_1,t_2,t_1) = 2\delta-2(1-\delta) = 4\delta-2$$
$$Eu_s(s_2,t_2,t_1) = -\delta-(1-\delta) = -1$$

➔ $Eu_s(s_1,t_2,t_1) > Eu_s(s_2,t_2,t_1)$ if and only if $\delta > ¼$

➔ The game has two BNE equilibria:

1. **$\{(s_1,t_2,t_1)$, $\delta > ¼$**} (when S believes that the incumbent party has at least 25% probability of being T_R , i.e. disposed to lose the college, S runs a national leader and T answers accordingly to its character)
2. **$\{(s_2,t_2,t_1)$, $\delta < ¼\}$** (S continues to present the local candidate and T reacts as before, if S believes that the incumbent party has at least 75% probability of being T_D , i.e. she is resolute to keep the college seat)

Same example with different payoffs (i.e., Nature defines M types)

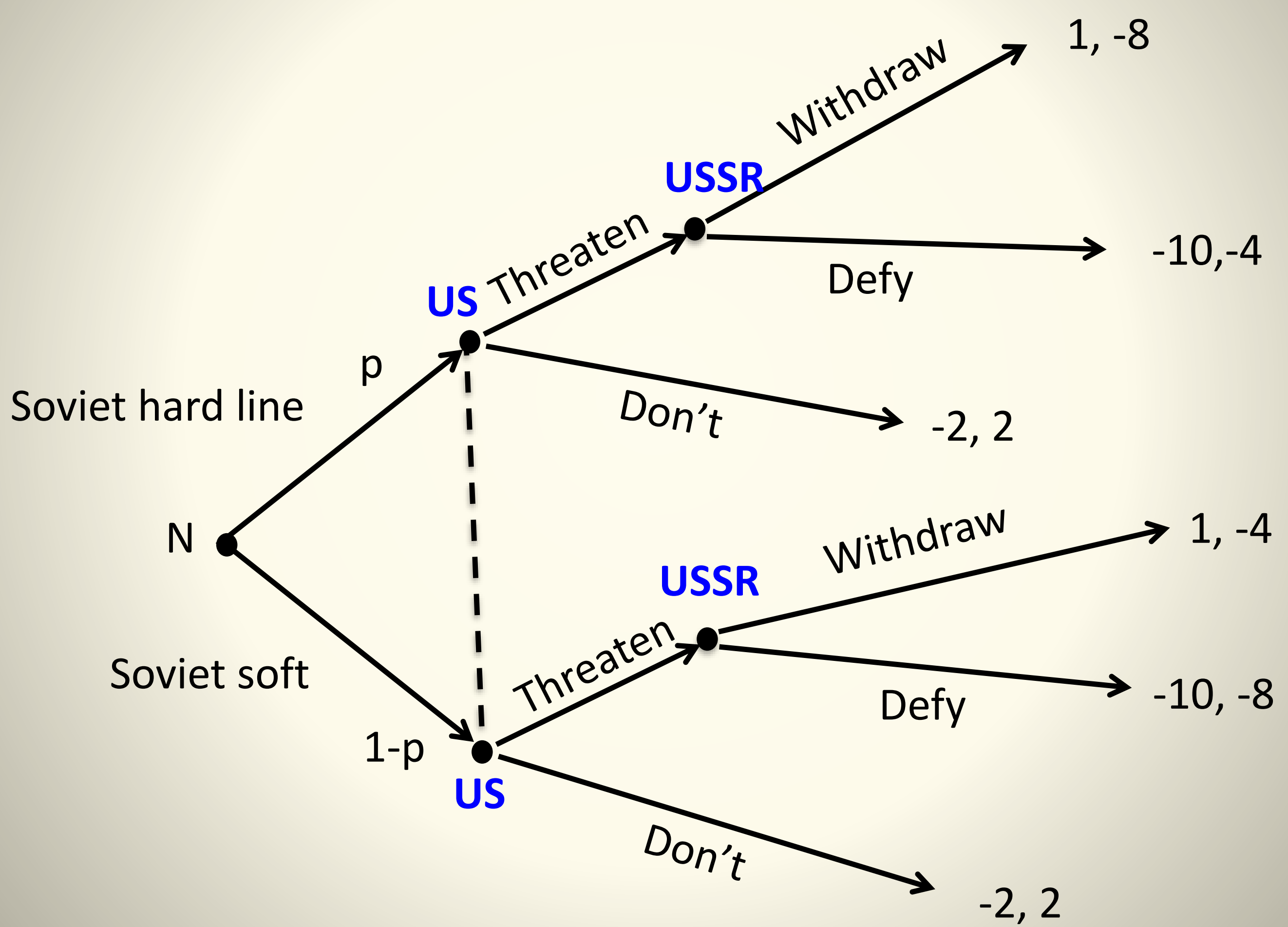


Two BNEs: (in, c, a) with y>1/2 and (out, c, a) with y<1/2

The Cuban Missile Crisis 1962

- In late summer and early fall of 1962, USSR started to place medium- and intermediate-range ballistic missiles in Cuba. First time that USSR had ever attempted to place their missiles and nuclear weapons outside Soviet territory
- In October an American U-2 “spy-plane” took photographs of it
- How US should respond to it?
- First option: “if you defy us, there will be nuclear war!”
- Kennedy’s estimate of a nuclear war (i.e., the value of p in the game below) was in the range from 1/3 to ½ (according to his published memories)

The Cuban Missile Crisis 1962



The Cuban Missile Crisis 1962

- Two BNE:
- (Don't; Defy, Withdraw) with $p>3/11$ is a BNE
- (Threaten; Defy, Withdraw) with $p<3/11$ is a BNE
- However $p<3/11$ (i.e., 0.27) is **below** the lower end of Kennedy's estimate (from 1/3 to ½, remember)
- Therefore, the simple bald threat: "*if you defy us, there will be nuclear war!*" is too large, too risky, and too costly for the US to make

The Cuban Missile Crisis 1962

- So the US should surrender (as implied by the other possible BNE: (Don’t; Defy, Withdraw) with p>3/11)?
- Not necessarily (and indeed this equilibrium did not materialize after all…)
- A *probabilistic threat* or *brinkmanship strategy*
- With a probabilistic threat, one players says to the other: “*if you don’t comply, there is a risk that something very bad for you will happen. By the way, it will also be very bad for me, but later I will be powerless to reduce that risk*”

The Cuban Missile Crisis 1962

- **Brinkmanship** is the creation and control of a suitable risk of this kind. It requires **two apparently inconsistent things**
- First: you must let matters get enough out of your control that you will not have full freedom after the fact to refrain from taking the dire action, and so your **threat will remain credible**
- Second: you must **retain sufficient control** to keep the risk of the actions from becoming too large and your threat too costly
- Such “controlled lack of control” looks difficult to achieve, and it is…the real difficulty is not how to lose control, but how to do so in a controlled way

The Cuban Missile Crisis 1962

- No time to discuss it unfortunately, but if you are interested about it, take a look at the book: Avinash K. Dixit, David H. Reiley Jr., Susan Skeath (2009). *Games of Strategy* (Third Edition), W. W. Norton & Company, chapter 15

Home exercise
(Nature defines player 2 types)