Game Theory

10 – Games of Incomplete information:
dynamic games with updating of players' beliefs

Review of lecture nine

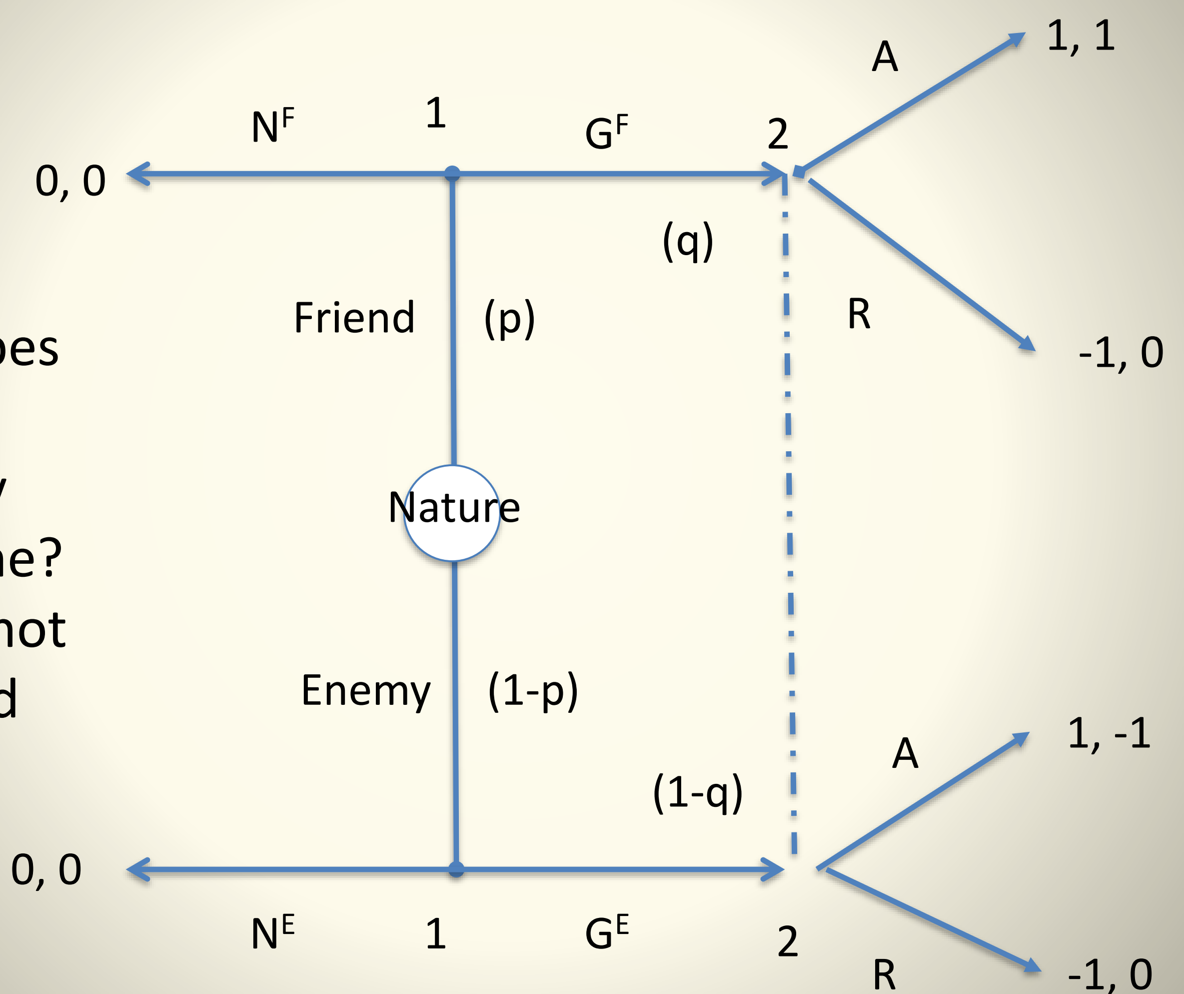
- Incomplete information, extensive forms and the role of Nature

Bayesian games with updating of beliefs

- Each type of player 1 has 2 strategies available (offering or not offering a gift)…
- …while player 2 has 2 strategies available at his/her information set (i.e., accepting or not accepting the gift)
- This game is a dynamic one where player 2 **can update his/her beliefs** about player 1 type according to what player 1 does…

An application

How many players and types of players?
Do you see any proper subgame?
If not, you cannot apply backward induction!



Bayesian games where the updating of beliefs is possible

- There is a difference, in this game, between **p** (i.e., the initial belief about player 1's type) and **q** (player 2's updated belief about player 1's type, **after that player 2 observes** the strategy of player 1)
- Here **q** is the probability that a Friend **is giving you** a gift
- For example, suppose that player 1 behaves according to strategy (N^F, G^E)
- As a result, according to such strategy by player 1, player 2 now expects a gift from an enemy with (an updated) probability equal to 1, i.e., p=q=0

Bayesian games where the updating of beliefs is possible

- In general, player 2 update always his belief about player 1's type, **conditional on arriving at player 2's information set - given player 1's strategies** (that is, conditional on receiving a gift in our example)
- How to include such a possibility into an equilibrium?

A Perfect Bayesian Equilibrium

- PBE is a solution concept that **incorporates** *sequential rationality* and *consistency of beliefs*
- *Sequential rationality* requires that players maximize their payoffs from each of their information sets (**on** or **off** the equilibrium path! More on this later…)
- How to reach that? *Consistency of beliefs*! In a PBE player's updated beliefs should be **consistent** with Nature's probability distribution **and** other player's strategy
- In general consistency between Nature's probability distribution (**p** in the previous example), player 1's strategy, and player 2's updated belief (**q** in the previous example) can be evaluated by using **Bayes rule**

Bayes rule

- Bayes rule gives the **conditional probability** of an event when another event has been observed, i.e., it gives us a criterion to determine **how new information should change our beliefs about a given event**

Bayes rule

- Let p(A) and p(B) two a-priori probability of the different events A and B
- Let us write **p(A|B)** the probability of the event A when B has been observed
- **Bayes rule is a formula for determining p(A|B)**
- More formally:
- $p(A|B) = (p(A)\ p(B|A))/[(p(A)\ p(B|A) + p(\neg A)\ p(B|\neg A)]$

where: p(A) is the a priori probability of A before occurring B, p(B|A) is the conditional probability of B given A, p(¬A) is the a priori probability of ¬A and p(B|¬A) is the conditional probability of B given the event “not-A”

Bayes rule

- **An example:**
- You are on the train and you want to understand if the person sitting next to you is a centre-right voter
- You know a priori that 54% of the Italian citizens are centre-right voters (46% centre-left)
- Now the person sitting next to you open a newspaper. You know that the 35% of centre-right voters read that newspaper (while it is read by 65% of centre-left voters)
- Which is your update belief that the person sitting next to you is a centre-right voter?
- $p(CR|N) = p(CR)\ p(N|CR)/[(p(CR)\ p(N|CR)+p(CL)\ p(N|CL))] =$

$=.54*.35/(.54*.35+.46*.65) = .387$

Bayes rule

Bayes rule:

- More in general, given (h_1, h_2,…, h_n) a set of mutually exclusive and exhaustive events compatible with the event k, then:

$$p\left(h_1 \middle| k\right) = \frac{p\left(h_1\right) p\left(k \middle| h_1\right)}{\sum_{i=1}^{n} p\left(h_i\right) p\left(k \middle| h_i\right)}$$

Bayes rule

- Going back to previous game:
- Suppose that the probability to meet a Friend determined by Nature is ½
- Let's further suppose that the two types of Player 1 adopt the following strategy (N^E,G^F)
- Before that strategy, p(FRIEND)=1/2. Now the update probability of p(FRIEND|G) is…
- p(FRIEND|G) = (p(FRIEND) p(G|FRIEND))/[(p(FRIEND) p(G|FRIEND) + p(ENEMY) p(G|ENEMY)]…that is:
- p(FRIEND|G) = (0.5 * 1)/(0.5 * 1 + 0.5 * 0) = 1

A Perfect Bayesian Equilibrium: definition

- Consider a **strategy profile** for the players (i.e., types), as well as **beliefs over the nodes at all information sets**. These are called a **PBE** iff:

1) each player's strategy specifies optimal actions, **given the strategies of the other players and her beliefs** (this is the same requirement of a BNE!)

2) the beliefs **are consistent** with Bayes rule *wherever possible* (?!? Be patient and you will understand it…)

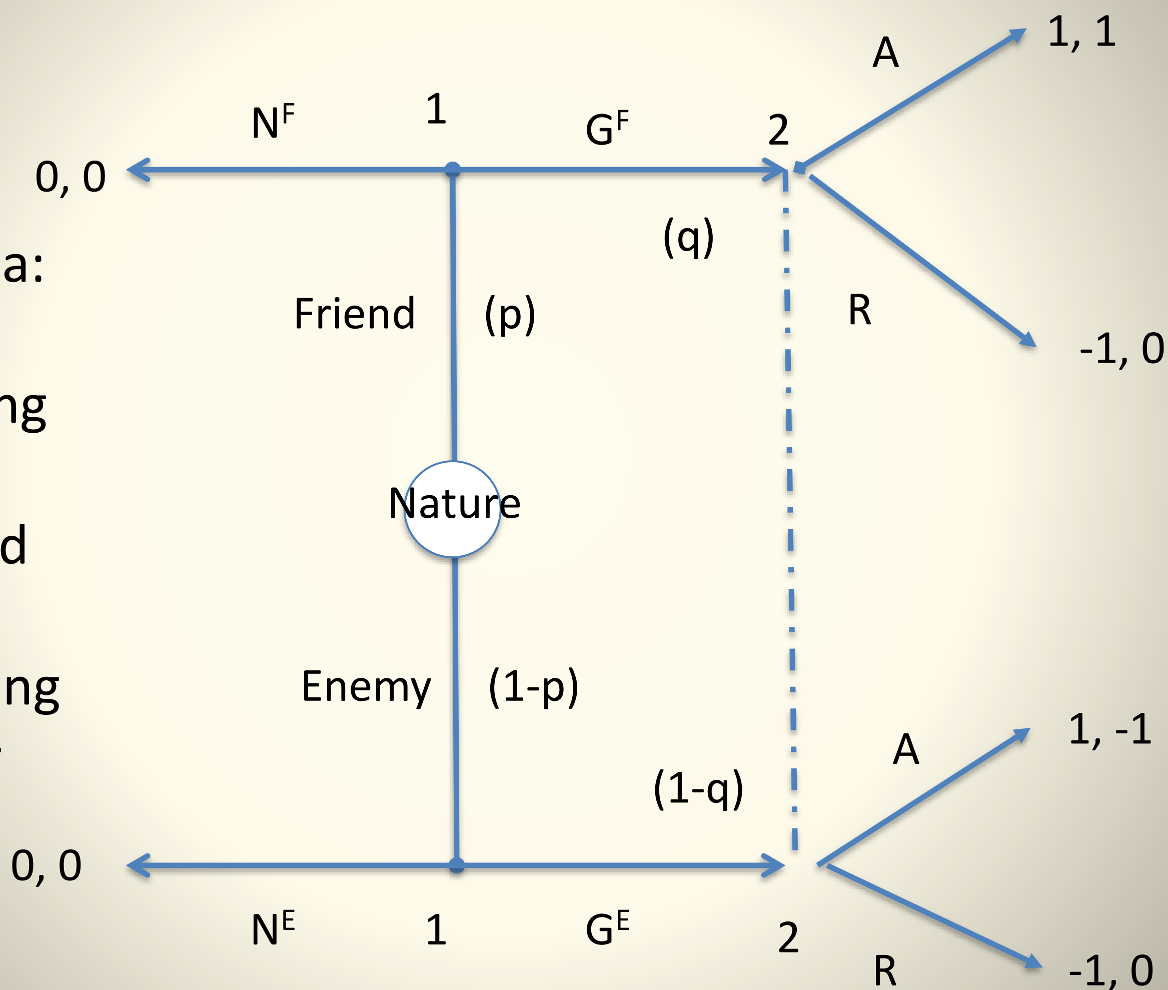
- In essence a PBE is a **coherent story** that describes **beliefs and strategies** in a game

A Perfect Bayesian Equilibrium: how to find it!

- Two additional terms are useful: we call an equilibrium as a **separating** one if all the types of a player behave differently
- We call an equilibrium as a **pooling** one if all the types behave the same

An application

There are **four** potential equilibria: two <u>separating</u> equilibria (featuring strategy $G^F N^E$, or strategy $N^F G^E$) and two <u>pooling</u> equilibria (featuring strategy $N^F N^E$, or strategy $G^F G^E$)



A Perfect Bayesian Equilibrium: how to find it!

– **Steps for calculating PBE:**

- Starts with a strategy for player 1 (in this case 2 strategies for the 2 types of player 1)
- If possible, calculate updated beliefs (**q** in the example) for player 2 by using Bayes rule. In the event that Bayes rule **cannot be used**, you must **arbitrarily select an updated belief**; here you will generally have to check **different potential values** for the updated belief with the next steps of the procedure;
- Given the updated beliefs, calculate player 2's optimal action
- Check whether player 1's strategy is a best response to player 2's strategy. If so, CONGRATULATIONS: you have just found a PBE!

An application

Let's apply our procedure:

Separating with N^F G^E:

- given this strategy for player 1, it be must be that q|G=0 (Bayes rule!). Thus, player 2's optimal strategy is R. But then the enemy type of player 1 would strictly prefer not to play G^E given that is not the best reply to R! The best reply to R would be for the enemy type N^E !
- Therefore, there is **no PBE** in which N^F G^E is played

An application

Separating with G^F N^E:

- given this strategy for player 1, it be must be that q|G=1 (Bayes rule!). Thus, player 2's optimal strategy is A. But then the enemy type of player 1 would strictly prefer to play G^E rather than N^E

✓ Therefore, there is **no PBE** in which G^F N^E is played

An application

Pooling with G^F G^E:

- here Bayes rule requires that q|G=p, so player 2 optimally selects A iff q=p>1/2. When q=p>1/2 there is therefore a PBE in which q=p and (G^F G^E, A) is played - PBE: (G^F G^E, A) , q=p; p>1/2
- **Why an equilibrium?** Given the strategy (G^F G^E) played by player 1, the best reply for player 2 to that GIVEN the belief specified (q=p; p>1/2) is A. And given the strategy adopted by player 2 (A), the strategy (G^F G^E) is the best reply to that for both players!

An application

Pooling with G^F G^E:

- On the other hand, in the event that $q=p<1/2$, player 2 must select R, in which case neither type of player 1 wishes to play G in the first place

✓ Thus there is **no PBE** of this type when $q=p<1/2$

An application

Pooling with G^F G^E:

- **But what will happen if q=p=1/2?**
- Then player 2 will be indifferent between playing A or R, so he will be mixing
- That implies looking for a mixed strategy PBE. That’s something we won’t discuss in this course

An application

Pooling with N^F N^E:

- In this case Bayes rule **does not determine** q
- Why? Cause in this case both types of player 1 play N, and player 2 **cannot update** q according to Bayes rule, given that G is not played and his information set is not reached **on the equilibrium path**!!!

An application

Pooling with N^F N^E:

- Still, regardless of player 1's strategy, player 2 will have some updated belief q at his information set
- This number has **meaning** even if player 2 believes that player 1 adopts the strategy N^F, N^E
- In this case, q represents player 2's belief about the type of player 1 when the "surprise" of a gift occurs (i.e., **off the equilibrium path**)

An application

- **Pooling with N^F N^E**:
- What would do player 2 in this eventuality? She will estimate her expected utility by playing either A or R given p
- This would lead to opt for R, iff $q<1/2$
- In other words, in order for R to be chosen, player 2 must have a sufficiently **pessimistic belief** regarding the type of player 1 after the "surprise" in which a gift is given
- Strategy R is therefore optimal as long as $q<1/2$. But given strategy R, N^F N^E is a best response to that by the two types of player 1!
- Note that without specifying what player 2 would do in the (surprising) eventuality she gets a gift, you cannot check if N^F N^E is actually an optimal strategy for both types of player 1!

An application

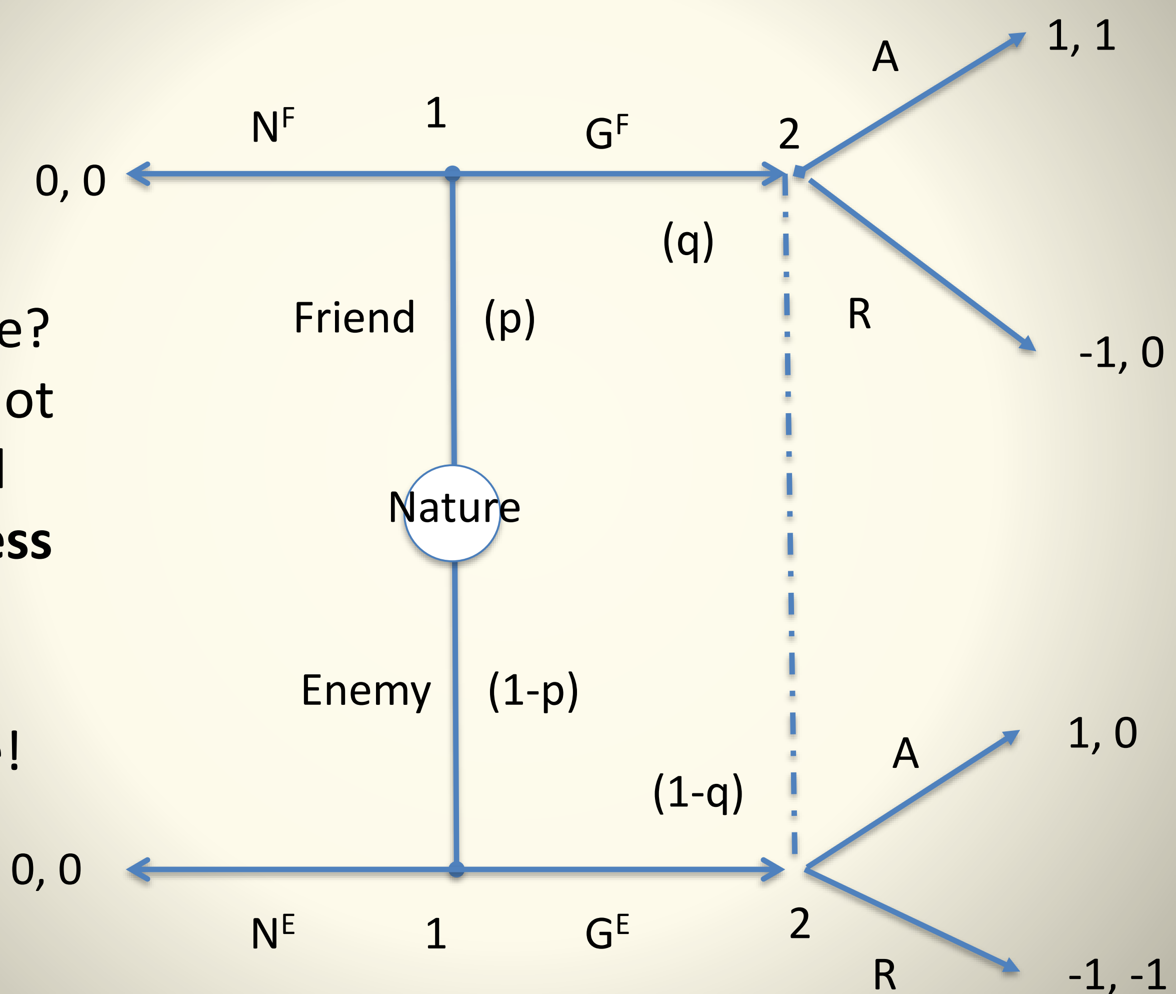
- **Pooling with N^F N^E**:
- Thus, for q<1/2 there is a PBE in which player 2's belief is p and the strategy profile (N^F N^E, R) is played
- In this equilibrium player 2 believes that an **eventual (off-the-equilibrium path) gift signals** the presence of the enemy (a misanthrope?)
- PBE: (N^F N^E, R) , q<1/2
- On the other hand, if q>1/2 player 2 would select A. But then bother types of player 1 would have an incentive to switch their strategy! No PBE!

Another example

- Consider once again **the gift game**. However, in this variant of the game player 2 always prefer opening gifts than not opening it (*satisfying curiosity is a lovely gift by itself!*) that is…

The Gift Game part 2

Do you see any proper subgame? If not, you cannot apply backward induction...**unless** one specific circumstances applies, as here!

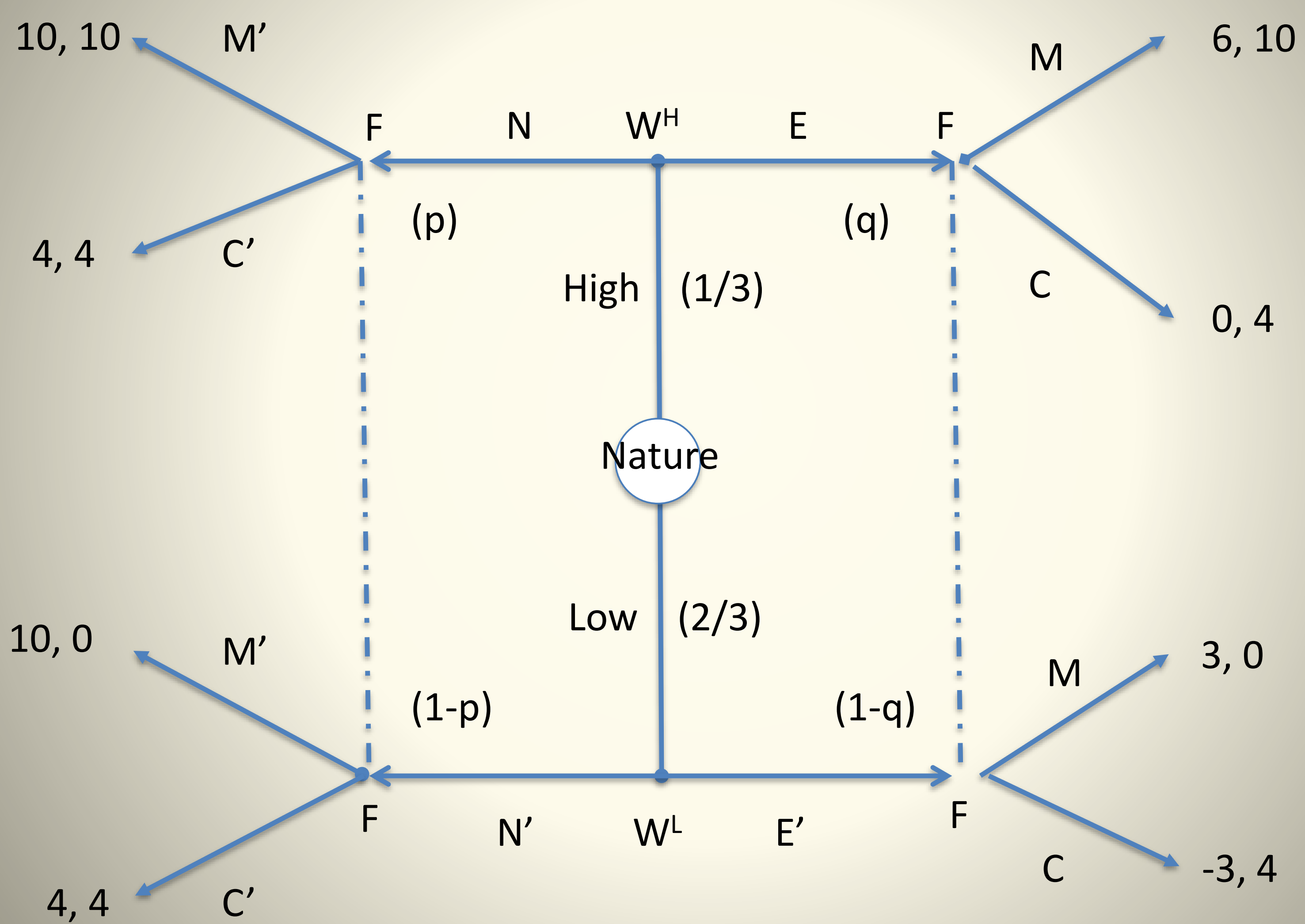


Only one BNE: (G^F G^E, A) , q=p; for any value of p

Job-Marketing Signaling

- **The signaling role of education**: which role of **formal education** in the **marketplace**?
- A worker (W) and a firm (F). The worker can be of **two types**: **high** or **low type**. Firm must decides whether to employ the worker in an important managerial job (M) or in a much less important clerical job (C). M produces a benefit of 10 to both types of worker, however they have **different education costs** (in terms not only of monetary costs, i.e., grants, etc.; but also in terms of opportunity costs): the high type to get an education must pay 4 units of utility, the low type 7. C produces a benefit of 4
- Importantly, education is of **no direct value to the firm**; the firm's payoff does not depend on whether the worker gets an education, but **only** on the intrinsic type of the worker

Job-Marketing Signaling



Job-Marketing Signaling

- Each type of worker has 2 strategies available (getting or not an education), while the firm has 2 strategies available at each of its 2 information sets (i.e., always offering a managerial job; always offering a clerical job; etc.)
- The initial system of beliefs of the Firm given by Nature is: High Type=1/3; Low Type=2/3

Job-Marketing Signaling: comments

- Two PBNE

1. the first one is (EN’, MC’, p=0, q=1)
2. the second one is (NN’, CC’, p=1/3, q<2/5)

✓ Why p=1/3? Bayes Rule!

p(high|N) = (p(high) p(N|high))/[(p(high) p(N|high) + p(low) p(N|low)]…that is: p(high|N) = (1/3 * 1)/(1/3 * 1 + 2.3 * 1) = 1/3

Job-Marketing Signaling: comments

- **Insights:**
  - **First**: the only way for the high-type worker to get the job that she deserves is **to signal her type** by getting an education. Otherwise the firm judges the worker to be a low type
  - **Second**: the value of education as a signaling device depends on the types' **differential education costs**, not on any **skill enhancement** that education deliver
- That is…to the extent that highly productive people are **more likely** than less-productive people to get degrees, than rather than helping people become smart, universities exist merely to help people who are already smart **to prove** that they are smart!

Job-Marketing Signaling: comments

- Finally, and with respect to the second PBE:

if firms are **enough pessimistic** about the chance to meet a high quality type when they observe the unexpected signal of “education” (i.e., they are quite pessimistic about the ability of the **educational costs to discriminate among types** perhaps because universities are not able to discriminate among such types…), both workers (including the high type) will not have any incentive to get a degree

Job-Marketing Signaling: comments

- This also means that **highly demanding** Universities and Professors are making you a favour given that reinforce the signal you are sending to the market! (this is something impossible to make understand to several ppl…)
- Similarly, **free universities** risk to depress the credibility of the signals sent by getting an education (given that they greatly reduce the education costs, so everyone could get a degree; but then, why getting it if it is not useful ex-post as a signalling device? Indeed no PBNE with EE'!!!)

Job-Marketing Signaling: comments

- Note **how you should employ a game**:

1) you start with some assumptions (here two: a) different education costs across types and b) no value per-se of a degree for a firm);

2) you derive some equilibria according to such assumptions;

3) you derive some insights from such equilibria that could be empirically tested

Job-Marketing Signaling: comments

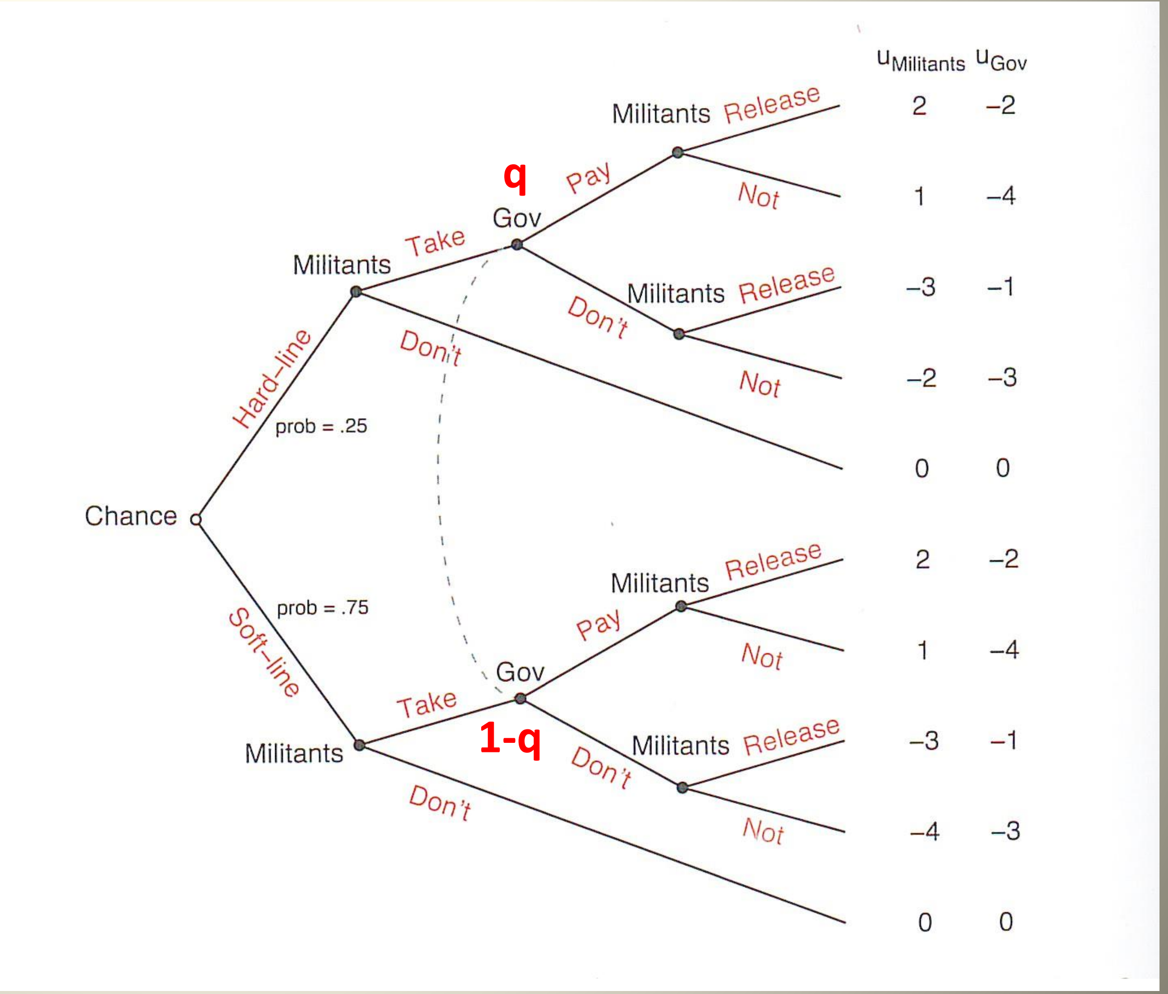
- Then, if the **empirical reality does not match the theoretical insights** you derive from the equilibrium (for example: it is not true that you have a lower % of university students in countries where university is free compared to countries where university is not free)…
- …you should go back to your assumptions and see if you can somehow relax them (perhaps for at least some degrees, the information you get at University is useful for firms after all)

Job-Marketing Signaling: comments

- Moreover let’s go back to the two possible PBNE
- A separating equilibrium is a PBNE where the actions in the equilibrium **reveal player type**
- A pooling equilibrium is a PNBEwhere the actions in the equilibrium **cannot be used to distinguish player type**
- This game is a typical **signaling-game**: a player can signal his/her own type by taking a **costly action** (i.e., getting an education and paying for it)

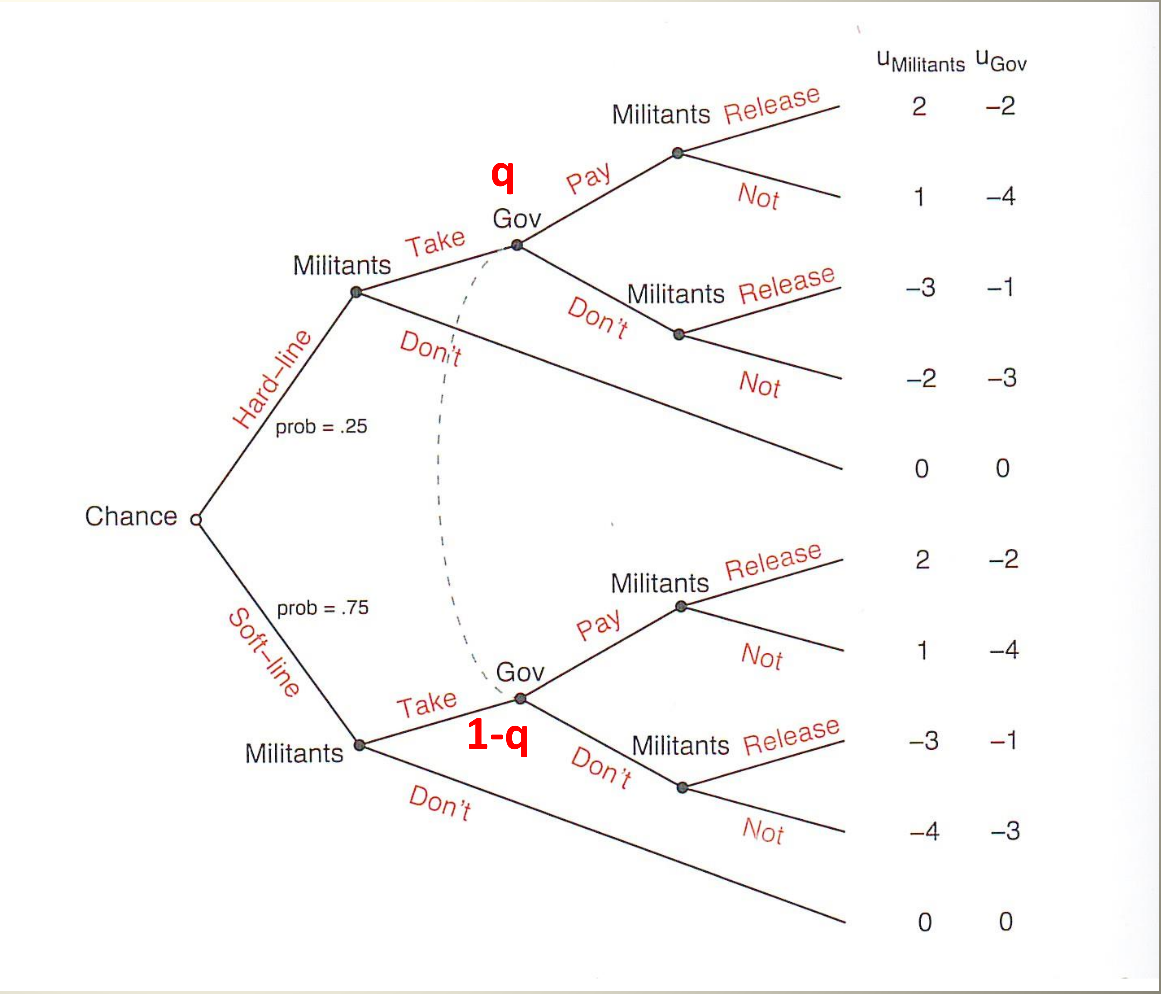
Let's go back to the ransom game

- Two players, three types
- 2 strategies available to Government (i.e., it is involved in just one information set)
- And how many strategies are available to each type of Militant?
- Each type of Militant is involved in three information sets, with 2 moves each
- Therefore: 8 strategies available to each of the 2 Militants type



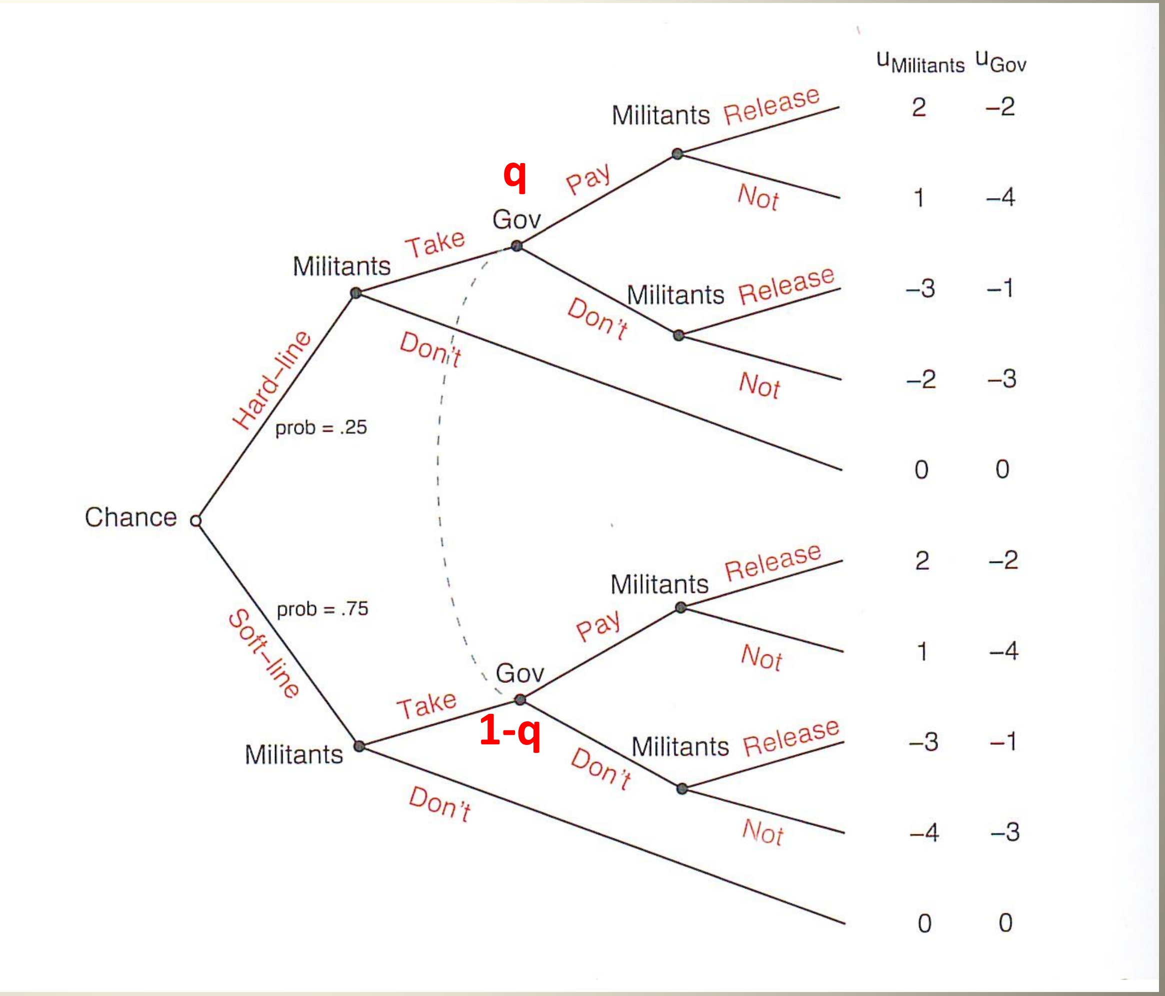
Let's go back to the ransom game

- Here **q** is the update probability that an hard-line militants is taking an hostage
- Remember that **q** can be updated only if the strategy played by at least one type of Militants reaches the information set of Government in equilibrium
- Otherwise, Bayes rule does not determine **q**
- **q** in this latter case represents Gov's belief about the type of Militants when the "surprise" of a kidnapping occurs (i.e., **off the equilibrium path**)



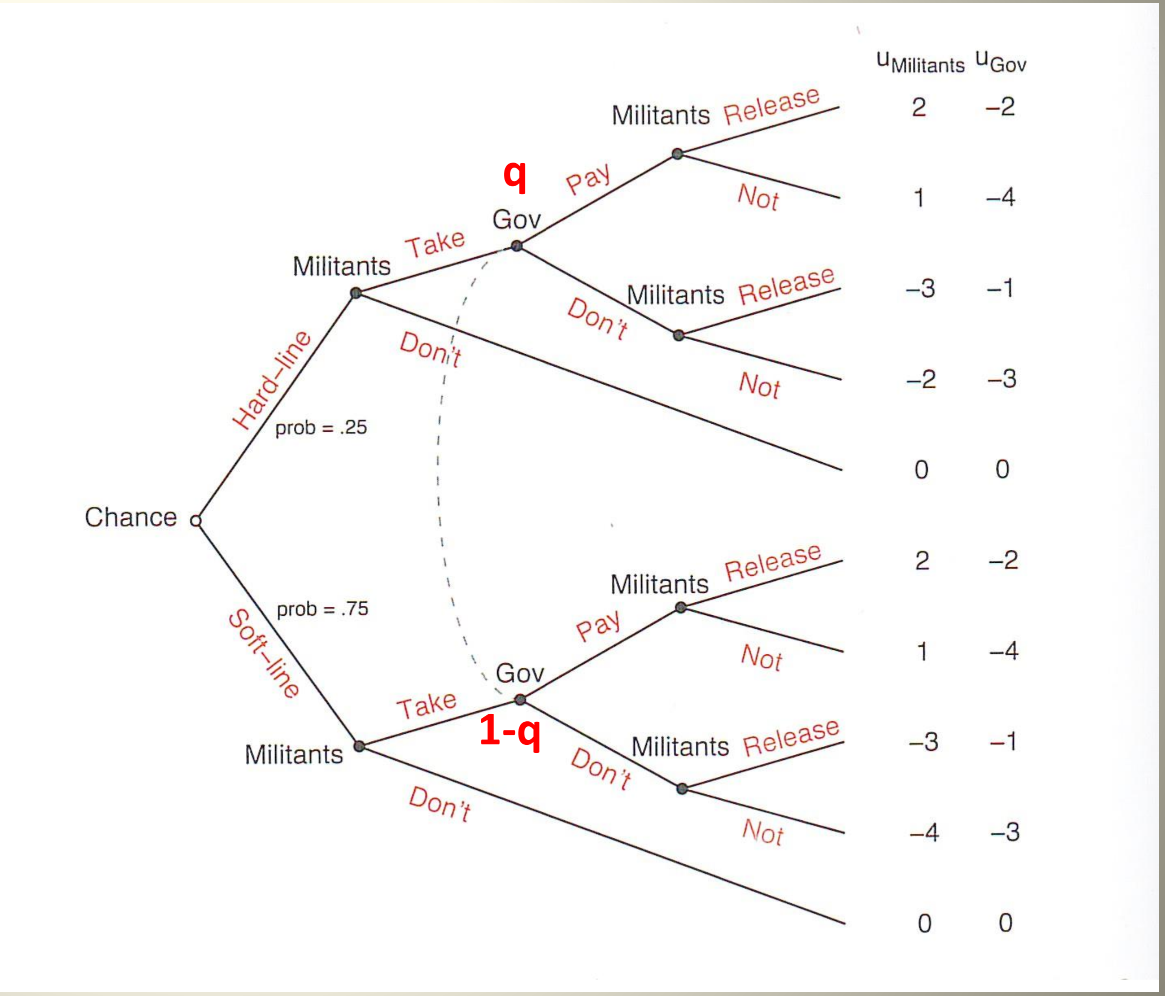
Let's go back to the ransom game

- Do you see any proper subgame?
- YES! 2 for each type of Militants
- Therefore you can apply backward induction in part of the game!!!



Let's go back to the ransom game

- 1 PBNE: (Don't, Release, Not Release; Don't, Release, Release; Don't) with p=0.25 and q<1/2



Przeworski's model of democratic transition

- A revised version of Przeworski's model (*Democracy and the Market*, 1991)
- Przeworski's models examine the strategic interaction between liberalizers within an authoritarian government and mobilizers within civil society
- The liberalizers make the initial move , deciding between opening up the political process (open) and maintaining the status quo (**stay tough**). If a decision to stay tough is made, the outcome is a strong dictatorship (SDIC)

Przeworski's model of democratic transition

- If a decision to open up the political process is made , civil society is given an opportunity to choose between entering into a compact (**enter**) with the state or organizing politically (**organize**). If civil society enters , the outcome is broad dictatorship (BDIC)
- Given a decision to politically organize, the liberalizers must decide whether to further political **reforms** or to **repress** the organized political activity
- If the liberalizers allow further political reform, political transition to democracy (**Transition**) is the outcome

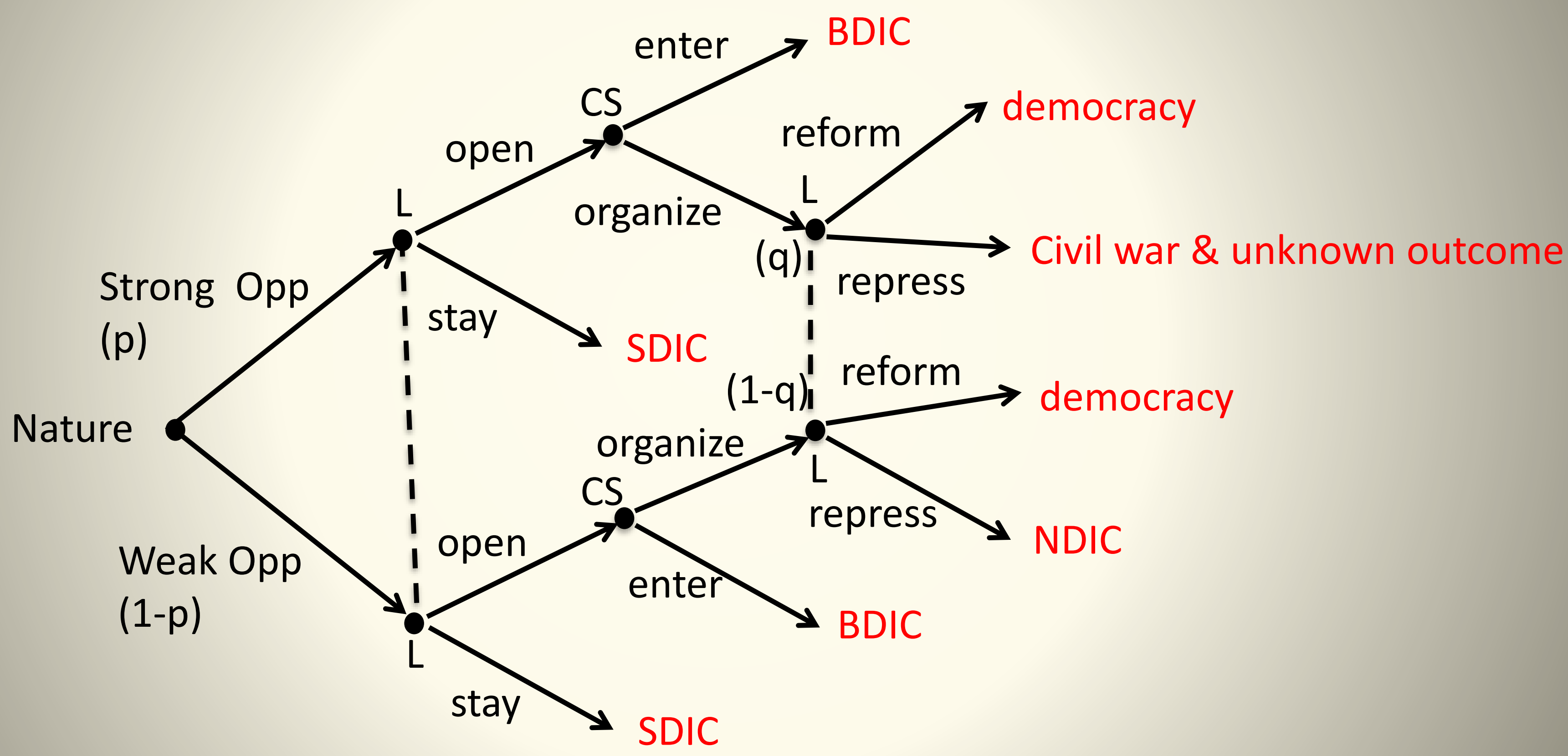
Przeworski's model of democratic transition

- However, if the liberalizers decide to repress, the outcome will depend on how **strong (or weak) are the mobilizers (the opposition)** within the civil society
- If the opposition is weak, then repression is successful, leading to a narrow dictatorship (NDIC)
- If the opposition is strong, then repression is unsuccessful, leading to a civil war with unknown outcomes (but surely a very costly one)
- Assuming that the civil war leads to the maintance of a «bloody» status-quo does not chance any of the results

Przeworski's model of democratic transition

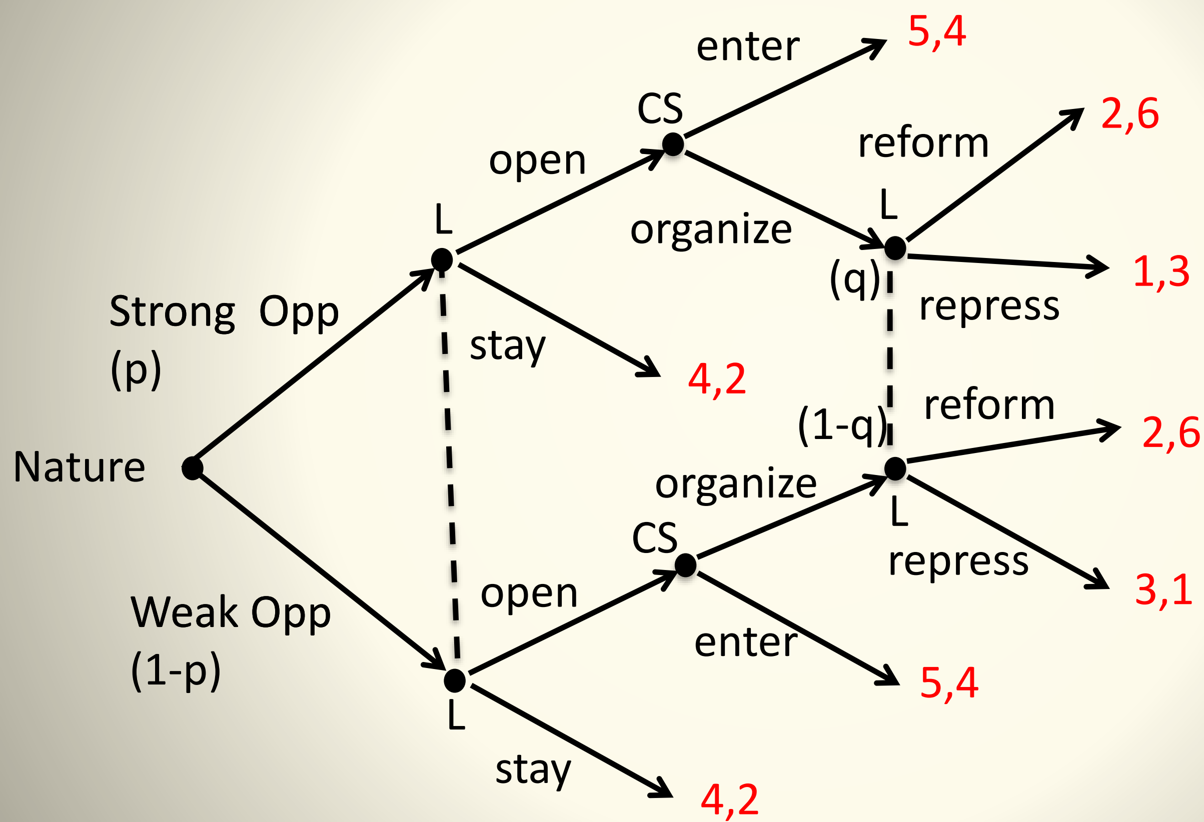
- Crucial aspect: the liberalizers **do not know the type of opposition they are facing**. It could be strong or weak…however, if they observe the opposition to «organize» they can update their priors

Przeworski's model of democratic transition



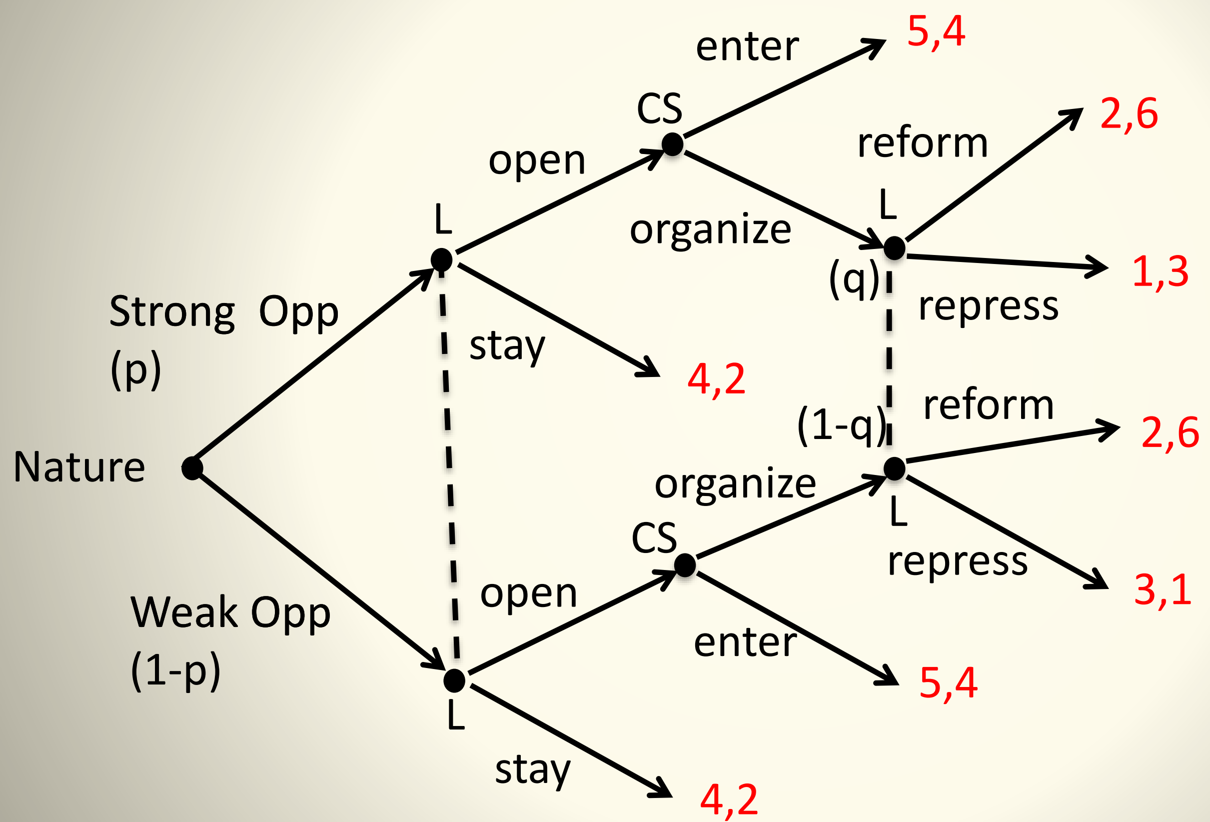
L = liberalizers; CS = mobilizers(opposition) within civil society

Przeworski's model of democratic transition



How many strategies are available to L?
How many strategies are available to the two types of CS?

Przeworski's model of democratic transition

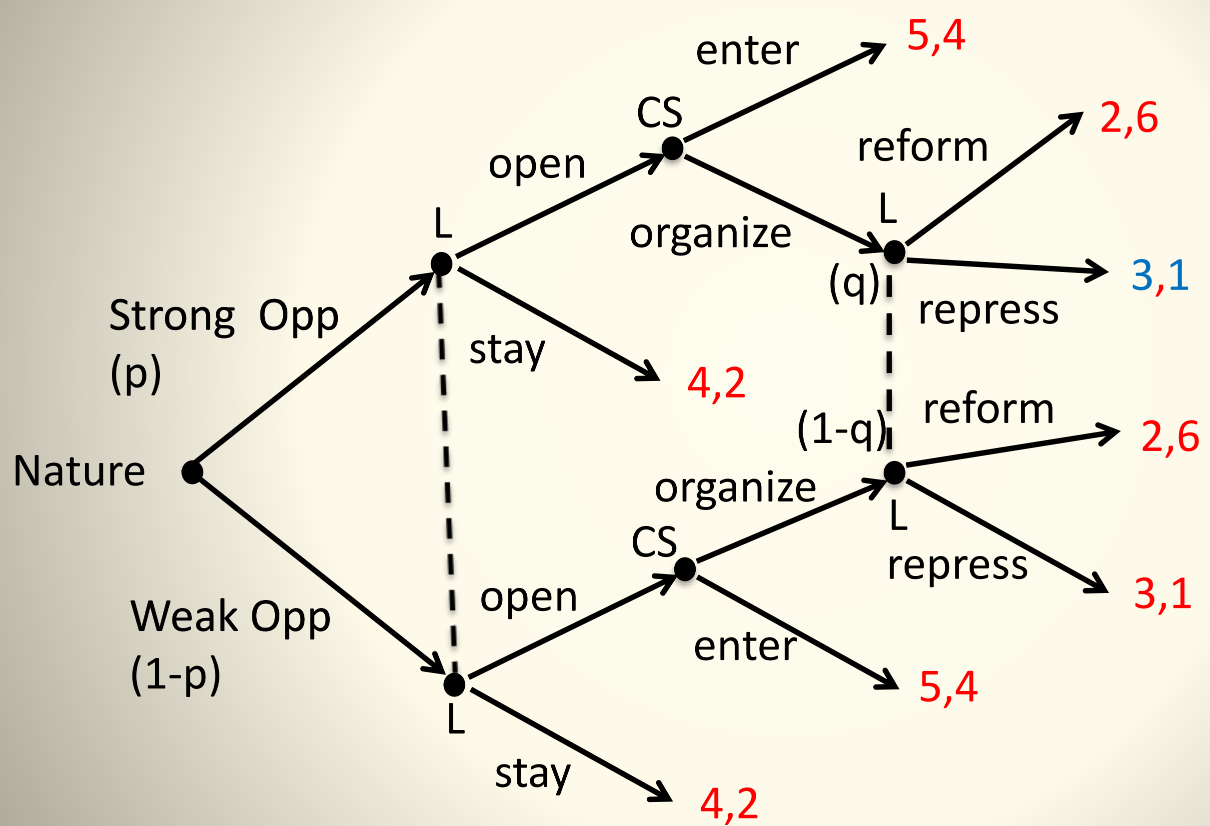


2 PBNE! 1) (Stay, Reform; Organize; Organize) with q>1/2; 2) (Open, Repress; Enter; Enter) with q<1/2

Przeworski's model of democratic transition

- What is going to happen if we assume that the civil war leads to the maintance of a «bloody» NDIC? The game becomes trivial! Why?

Przeworski's model of democratic transition



Przeworski's model of democratic transition

- Therefore, the game does not allow in equilibrium a transition to democracy? To understand under which conditions that could happen, read Przeworski (1991)!

Some further dynamics

- Let's discuss two further game dynamics:

1. Cheap-talk games
2. Semi-separating equilibria

Signaling vs cheap-talk games

- As we already underlined, in the job-marketing game we saw earlier a player signals his/her own type by taking a **costly action** (i.e., getting an education and paying for it)
- However, a player can try to do the same by doing something else…like for example…**talking**!
- However note the difference! Talk per-se has no real opportunity cost, and the content of speeches can be very costly to verify

Signaling vs cheap-talk games

Examples:

- ✓ Your doctor tells you that you should go through an expensive MRI test
- ✓ Your investment analyst recommends you to buy stocks of a particular company.
- ✓ A lobbyist (expert on a particular topic) informs a politician about the current conditions in a given industry

Cheap-talk games

We assume that the speech making is **strategic**, that is, rational speakers choose their words to attempt to convince audiences to make one set of choices rather than another…

- …and rational listeners recognize the strategic nature of the speeches in decoding the **truth of falsity** of any arguments or claims the speakers may advocate
- As such, what ultimately matters is not how eloquent a speech may be, say, but **what information is succesfully transmitted** to listeners

✓ two different speeches that lead listeners to the same conclusions are strategically equivalent

Cheap-talk games

- Are there situations in which you would believe that whatever comes out of his/her mouth is the true, i.e., in which we observe in equilibrium "*information transmission*“?
- That is, we would like to search for **separating equilibria** where for example:
  - the doctor tells you to take the test only when necessary
  - your investment analyst recommends to buy only when the prospects of the firm are good
  - the lobbyist informs the politician about the true state of the industry

Cheap-talk games

The structure of the game

- First, nature chooses the *sender's type* (i.e., its private information on the true state of the world)
- Second, the *sender* learns her type and chooses a message
- Third, the *receiver* observes the sender's message, modifies his beliefs about the sender's type, and chooses a response
- An equilibrium requires that the players select *optimal moves* given the message(s) they have received and that they select *optimal messages* given their expectations about what moves will result from those messages

Cheap-talk games

An example

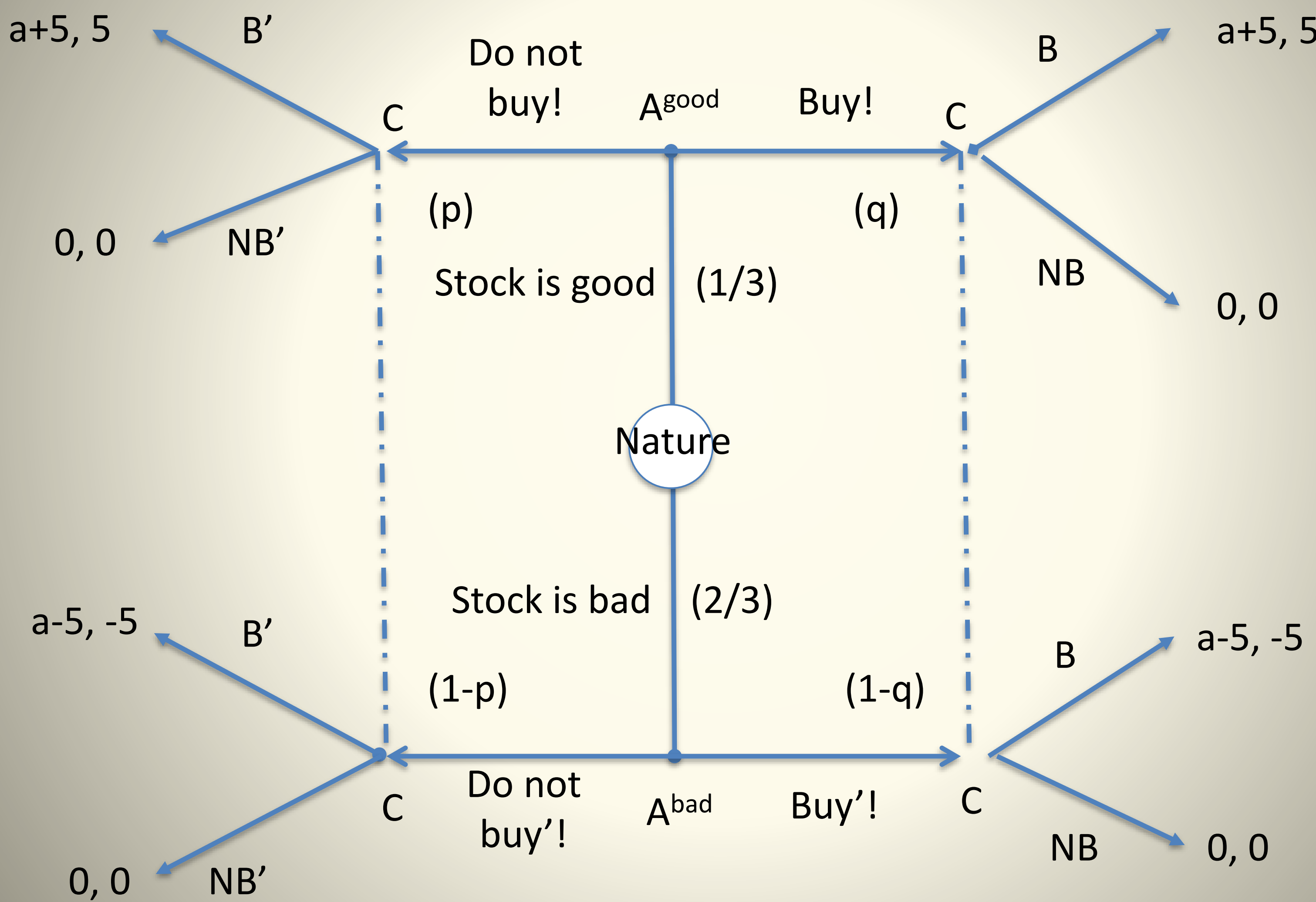
- Nature moves first determining the value of stock
- With probability 1/3 the stock is a good investment; with probability 2/3 no
- The value of the stock is only known by your investment analyst
- After determining the value of the stock for the client, the analyst decides to recommend/do not recommend to buy the stock
- The client then chooses whether to buy the stock, after receiving his investment analyst's recommendation/not recommendation of buying the stock

Cheap-talk games

The payoff of your investment analyst

- ✓ If the client buys the stock when such stock is good, the analyst's payoff is ($a + 5$)
- ✓ If the client buys the stock when such stock is not good, the analyst's payoff is ($a - 5$)
- ✓ If the client does not buy the stock, the analyst's payoff is *0*
- Notice that if a = 0, then the interests of the analyst and the client coincide, i.e., **preferences are aligned**
- We assume that the investment analyst has a bias towards buying the stock (she obtains a benefit equals to *a*, i.e., *a* is a *positive number*) even if it a bad stock

Cheap-talk games



Cheap-talk games

We solve it in the usual way!

- Let's check if we have a pooling equilibrium in which the analyst suggests to buy (or not to buy) regardless of the true state of the world (i.e., regardless of her own type)
- Do we have it? YES!
- PBNE: (Buy Buy', NB NB', q=1/3, p<1/2)
- p<1/2 can be for example 1/3 (i.e., the initial probability)
- That is, in this equilibrium the client maintain her prior belief (stock good=1/3) regardless the message of the analyst
- Similarly:
- PBNE: (Do not buy Do not buy', NB' NB, p=1/3, q<1/2)

Cheap-talk games

- In a cheap-talk game a pooling equilibrium, also known as "*babbling equilibrium*", is **always** a possibility
- In such equilibrium, a player does not condition her move on the message
- The message "babble" the receiver's type; it does not provide any information; therefore the receiver does not condition her move on the message and any message is "optimal" given that

Cheap-talk games

- Can we support some more information transmission in this game as an equilibrium?
- Yes!
- PBNE: (Buy Do not buy’, B NB’, q=1, p=0 iff *a*<5)
- If *a*>5 the analyst recommends to buy the stock, and this separating strategy profile cannot be supported as PBNE

Cheap-talk games

Intuition:

- The difference in the preferences of the client and the analyst, captured by parameter *a*, must be relatively small ($a < 5$) for a separating PBE to be supported, i.e., the two set of preferences should be at least partly aligned!
- Otherwise, only pooling (babbling) PBNEs are sustained, in which the analyst recommends the stock regardless of its future performance
- These are uninformative equilibria, since the uninformed client cannot infer any information about the stock quality from observing that his analyst has just recommended the stock
- We can then interpret **separating PBNEs as equilibria where information is transmitted from the informed to the uninformed player**

Cheap-talk games

Intuition:

- That is, for any credible information to be communicated through (costless) talk, two things at least must be true

1. First, it is in the speaker's interest to tell the truth, given that she is believed and the listeners act relative to that belief
2. Second, it is not in the speaker's interest to lie, given that she is believed and the listeners act relative to that belief

- If in any sistuation these two conditions are jointly satisfied, then prima facie it is plausible for the listeners to trust the speaker
- A wonderful paper:

David Austen-Smith (1992). Strategic models of talk in decision making, *International Political Science Review*, 13(1), 45-58

Cheap-talk games

- Any other possible equilibria? A perverse cheap-talk equilibrium?
- Yes but not very reasonable (why if the true state of the world is “good stock”, your analyst should advice you not to buy it?)!!!
- PBNE: (Do not buy Buy’, B’ NB, q=0, p=1 iff a<5)

Cheap-talk games

There are essentially two sorts of information that are possibile in this context

1. The first sort is ***technical*** (the game discussed above – i.e., when one player posseses some private information)

For a speaker to be able to persuade the listener to act in a particular way, it is clearly essential that the listener believe the speaker knows something that the listener does not (i.e., the speaker has private information and this private information is valuable for the listener)

2. The second sort of information concerns ***intentions***. Such speeches can be verbal threats or promises, or act as coordinating devices

Cheap talk & coordination

		Mr. Red	
		LEFT	RIGHT
Mr. Green	LEFT	(1 , 1)	(-1 , -1)
	RIGHT	(-1, -1)	(1 , 1)

- Here players' interests are perfectly aligned
- Not only focal-point to solve this game!
- Suppose Mr. Green says to Mr. Red: "I am going left" (n.b. **this is NOT a two-stage game**!)
- How Mr. Red will evaluate the credibility of such statement?

Cheap talk & coordination

- Mr. Red should ask himself 3 questions:

1. First, if Mr. Green is really going LEFT, would he want me to believe he is? Here, he would!
2. Second, if Mr. Green is really going RIGHT, might he wants me to believe he's going LEFT? Here, he would not!

So the message "I’m going LEFT" is **self-signaling**: Mr. Green wants to say it if and only if it is true!

3. If Mr. Green thought he had persuaded me that he’s going LEFT, would he have an incentive to go LEFT? Here, he would!

So the message is also **self-committing**: if believed, it creates incentives for the speaker to fulfill it!

<u>Cheap-talk beats focal points</u>!

Cheap talk & coordination

		Woman	
		Football	Opera
Man	Football	(3 , 2)	(1 , 1)
	Opera	(0 , 0)	(2 , 3)

- Here players' interests are partially aligned
- Suppose the Man says to the Woman: "Football"
- How the Woman will evaluate the credibility of such statement?

Cheap talk & coordination

1. If Man is really going football, would he want me to believe he is? Here, he would!
2. If Man is really going opera, might he wants me to believe he's going football? Here, he would not!

So the message is **self-signaling**: the Man wants to say it if and only if it is true!

3. If Man thought he had persuaded me that he’s going football, would he have an incentive to go football? Here, he would!

So the message is also **self-committing**: if believed, it creates incentives for the speaker to fulfill it!

In the case of only one player messaging, this could give that player a first-mover advantage!

Cheap talk & coordination

		Mr. Red	
		Cooperation	Defect
Mr. Green	Cooperation	(3 , 3)	(-1, 4)
	Defect	(4, -1)	(2 , 2)

- Any role for cheap talk in a PD? Here players’ interests are not aligned at all!
- Say Mr. Green says “I will cooperate; I expect you to do the same”

Cheap talk & coordination

		Mr. Red	
		Cooperation	Defect
Mr. Green	Cooperation	(3 , 3)	(-1, 4)
	Defect	(4, -1)	(2 , 2)

- This message is neither **self-signaling** (if he intends to defect, Mr. Green would like Mr. Red to believe he will cooperate), nor **self-committing** (Mr. Green will have no incentive to follow through on his promise, whether he expects Mr. Red to believe him)

Game of political entry

- You have two Left parties: L and LL
- L currently enjoys a monopoly in the political market for the left-side of the ideological spectrum
- LL is deciding whether to enter the political market or not, but LL does not know **how tough** a competitor L will prove to be
- Specifically, the loyalty of L's constituency can be high or low, implying a strong or a weak L
- If L is weak (L_w), LL can enter the political market and compete profitably; if it is strong (L_S), no
- However LL has only some priors about it, w/o being sure

Game of political entry

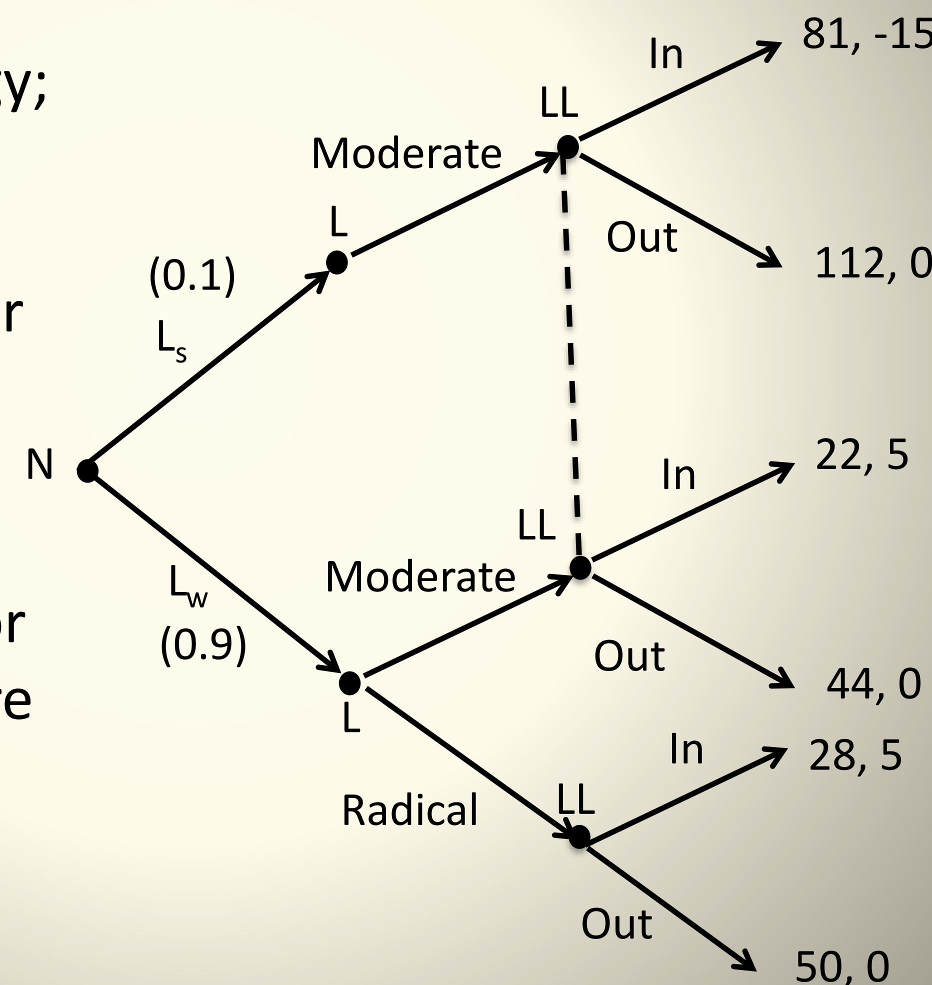
- We have a dynamic game
- L publishes a manifesto before an election. Let's suppose that a strong L will publish a moderate manifesto, cause it is sure that it won't lose the loyalty of its more radical constituency in its attempt to win the moderate voters
- A weak L can publish either a moderate or a radical manifesto
- It can in fact try to use its manifesto in the first stage to bluff LL into staying out by pretending it is strong by sending as a signal a moderate manifesto
- On the contrary, if L publishes a radical manifesto, LL will rightly infer that L is a weak party (and it is therefore challengeable)
- Of course LL is a strategic player and is aware of this possibility…

Game of political entry

There are two types of player L. L_s has one strategy; L_w has two strategies

LL is involved in two proper subgames. Four strategies available to LL

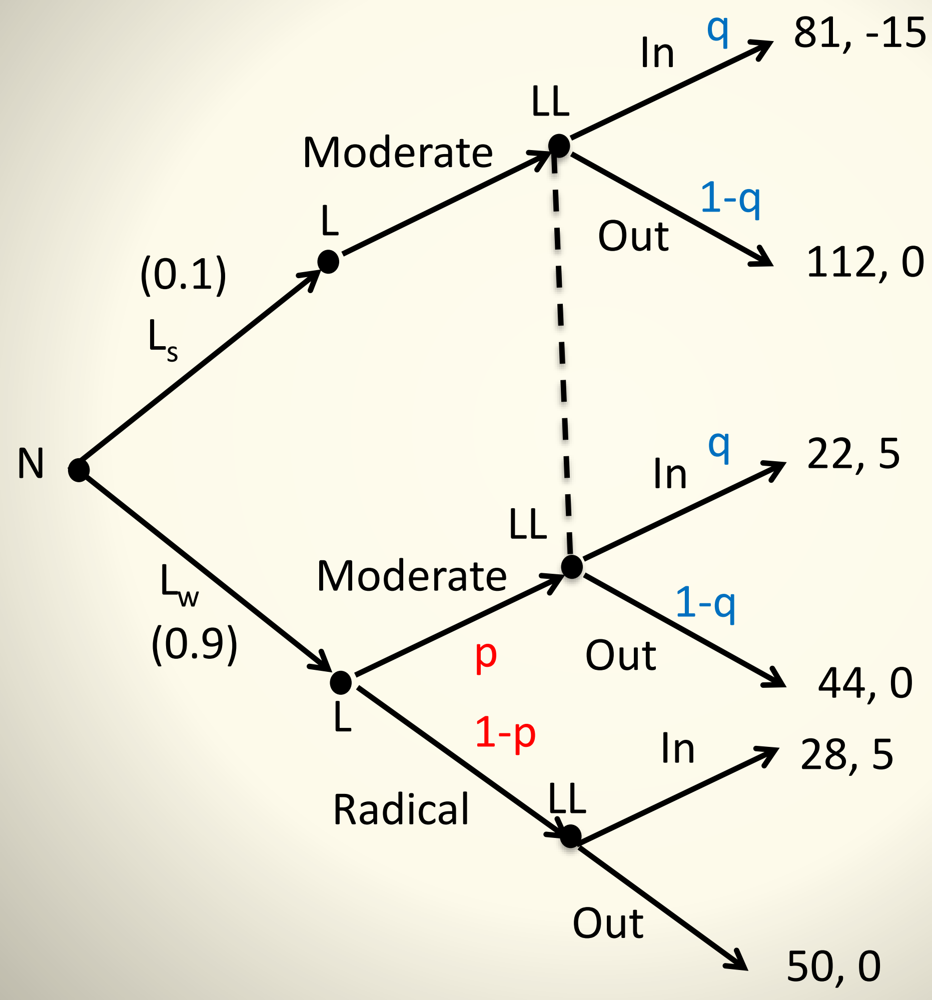
Here no separating (MR) or pooling equilibria (MM) are possible…



Game of political entry

- We can look therefore for a Mixed Strategies Equilibrium!
- First, we suppose L_w mixes Moderate and Radical with probabilities p and (1-p) respectively
- Similarly, LL mixes In and Out with probabilities q and (1-q)
- Let’s begin with the first point

Game of political entry



Game of political entry

Let's begin with the first point. We need to go back to Bayes rule!

Cause now the weak L mixes, and this affects what LL will think when it sees a Moderate platform…

- $Pr(L_w|Moderate)=Pr(L_w)*Pr(Moderate|L_w)/[Pr(L_w)*Pr(Moderate|L_w)+Pr(L_s)*Pr(Moderate|L_s)]=0.9*p/(0.9*p+0.1*1)$
- $Pr(L_s|Moderate)=Pr(L_s)*Pr(Moderate|L_s)/[Pr(L_s)*Pr(Moderate|L_s)+Pr(L_w)*Pr(Moderate|L_w)]=0.1*1/(0.1*1+0.9*p)$

Similarly…

- $Pr(L_w|Radical)=Pr(L_w)*Pr(Radical|L_w)/[Pr(L_w)*Pr(Radical|L_w)+Pr(Ls)*Pr(Radical|Ls)]=0.9*(1-p)/(0.9*(1-p)+0.1*0)=1$
- $Pr(L_s|Radical)=Pr(L_s)*Pr(Radical|L_s)/[Pr(L_s)*Pr(Radical|L_s)+Pr(L_w)*Pr(Radical|L_w)]=0.1*0/(0.1*0+0.9*(1-p))=0$

Game of political entry

- For having a mixed strategies equilibrium, L_w's p-mix must keep LL indifferent between its two pure strategies (i.e., (In,In) and (Out,In)) *coherently* with subgame perfection
- Therefore we need that the two EU are equal to each other. This happens when:

(-15*(0.1)/(0.1+0.9*p)+5*(0.9*p)/(0.1+0.9*p))+5*1=0+5*1

…that is p=1/3

Note that the original probability 0.1 of L being Strong was too low to deter LL from entering (i.e., playing In). LL's revised probability (i.e., Pr(L_s |Moderate)= 0.1*1/(0.1*1+0.9*p)) is now 0.1/0.4=0.25. Why?

✓ Precisely because the L_w is not always bluffing! If it were, the Moderate platform would convey no information at all. LL revised probability would equal 0.1 in that case; as a consequence it would play its pure strategy (In, In). But then we wouldn't have any equilibrium at all…

Game of political entry

- Let's now move to the second point
- For having a mixed strategies equilibrium, LL's q-mix must keep L_w indifferent between its two pure strategies (i.e., Moderate or Radical) – we don't care about L_S, cause it has just one strategy and it will always plays that!
- Therefore we need that the two EUs of L_w are equal to each other. This happens when:

(22*q+44*(1-q))=28, that is when q=8/11

That is, LL does not react always playing In against a Moderate move (cause if it would, once again we wouldn't have any equilibrium!)

As a result we have the following Mixed Strategies Perfect Bayesian equilibrium: (Moderate, Moderate with p=1/3; In with q=8/11, In)

Game of political entry

- In this equilibrium, the L types are only **partially separated**. The L_s always publishes a Moderate platform, but the L_w mixes and will also publish the Moderate platform one-third of the time
- If LL observes a Radical platform, it can be sure that L is weak; in that case it will always enter
- But if LL observes a Moderate platform, it does not know whether it faces a truly strong Left or a bluffing weak Left. Then also LL plays a mixed strategy, entering 8/11 of the time
- Thus, a Radical platform conveys full information, but a Moderate platform conveys only partial information about L's type
- Therefore this kind of equilibrium is labeled **semi-separating**
- A semi-separating equilibrium is a PBNE where the actions in the equilibrium convey some additional information about the players' types, but some ambiguity about these types remains